Hierarchical Ontology
and Knowledge Representation

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University of Maryland

October 26, 2019

Ontology

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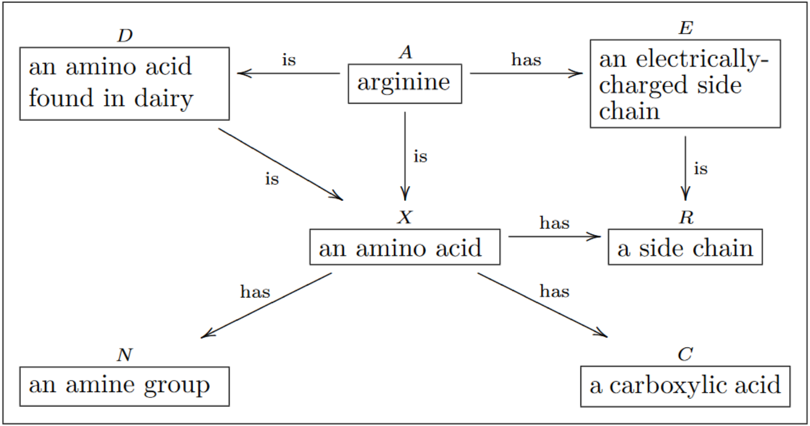

Picture taken from [OLOG]

Goals

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The goal is to define an rigorous and expressive notion of **Ontology**

This includes:

- A means of organizing data
-
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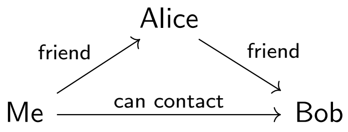

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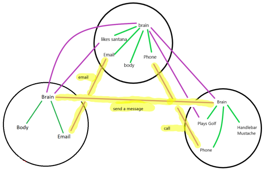

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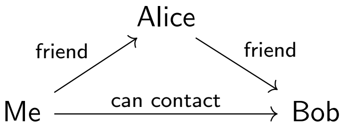

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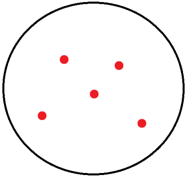

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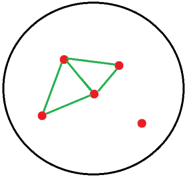

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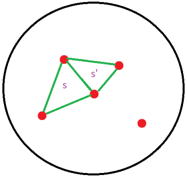

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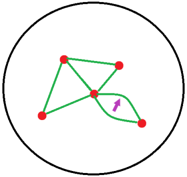

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Goal 1

An **Ontology** should organize data

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Hence the name "basic": just a framework

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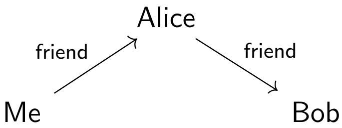

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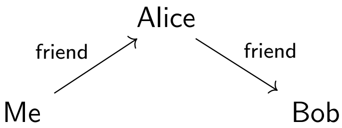

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This is fine in a Simplicial Set

Problem with composition in general

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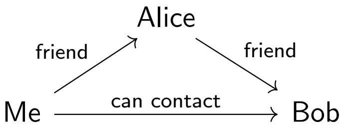

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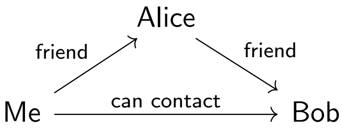

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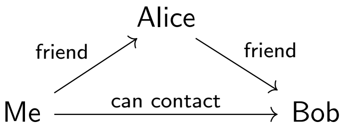

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but it is still somewhat ill defined

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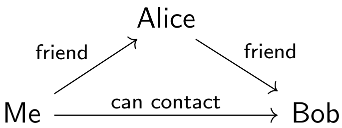

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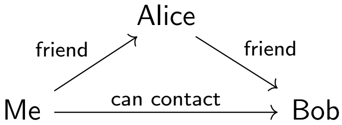

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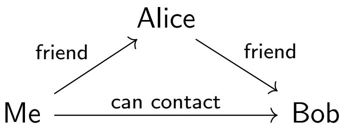

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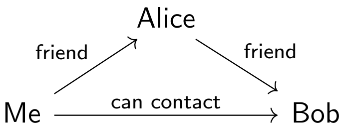

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- relations are undefined (what does “friend of” mean)
- composites are undefined
  (how does “friend of a friend” imply “can contact”)

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As humans, we have the ability to elaborate on concepts

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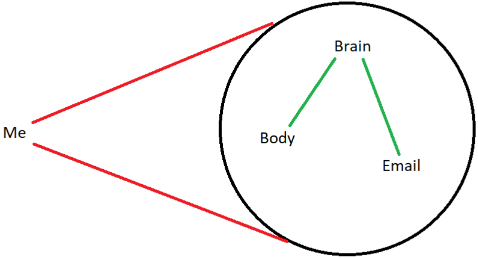

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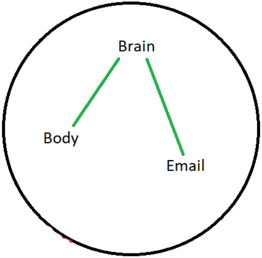

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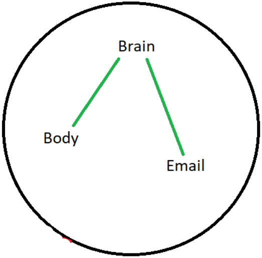

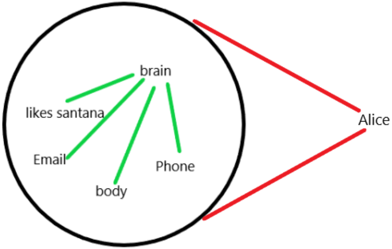

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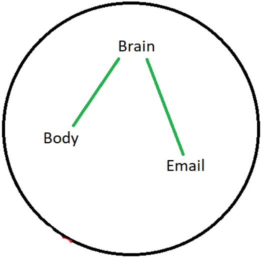

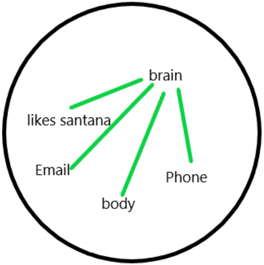

Elaboration of Relations

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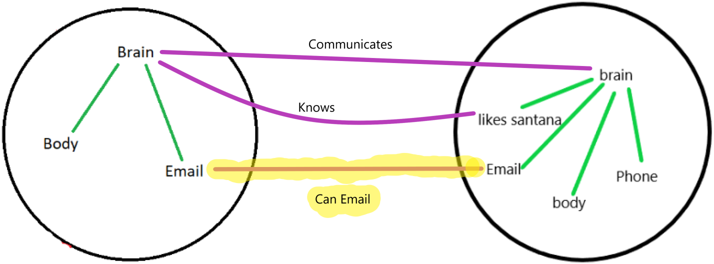
I can elaborate upon "Friend"

$$\text{Me} \xrightarrow{\text{friend}} \text{Alice}$$



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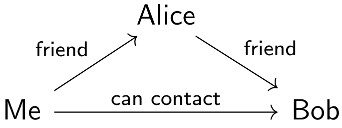
As humans, we have the ability to elaborate on concepts
Consider the proposed composition



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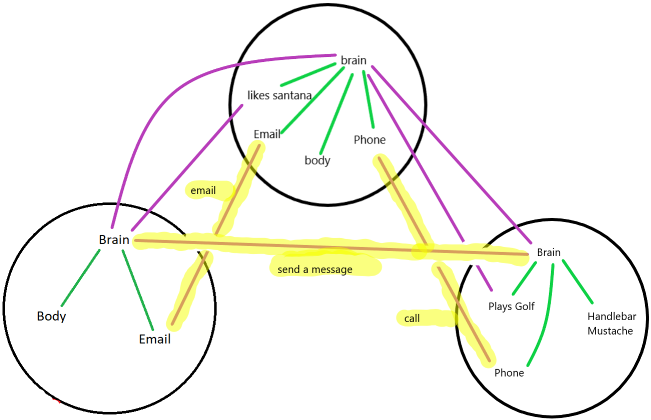

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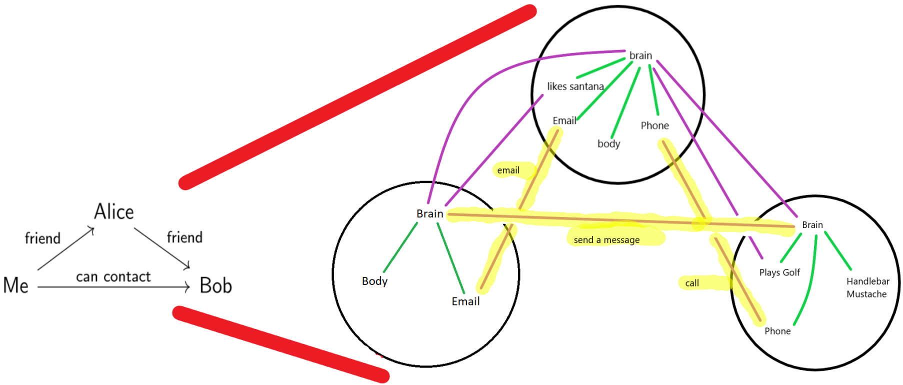

Expansion: Abstract ! Concrete

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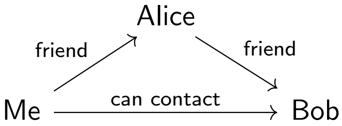

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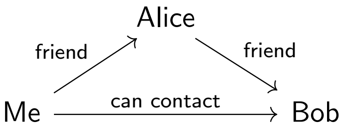

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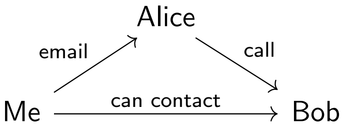

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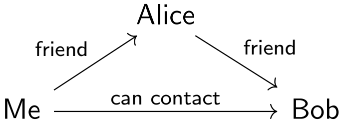

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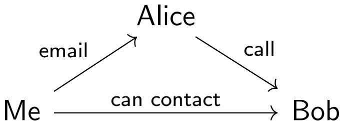

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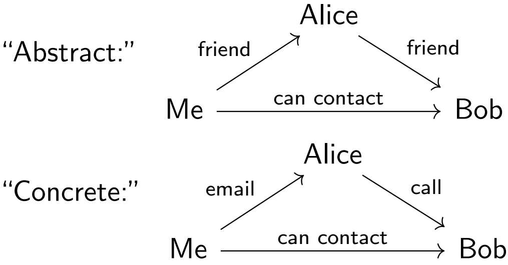

- Concrete ideas are part of these elaborations.
- Elaborations are subconcepts which "define" the idea

Open Sets: Global ! Local

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Goal 2

To define a process of elaborating upon data

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Consider the graph of topological spaces (bounded in cardinality)

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- These come with inclusions $J_0 : G_0$! G and $J_1 : G_1$! G

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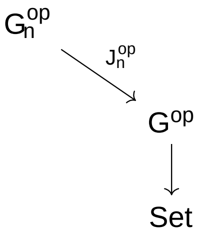

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$$G_n^{op} \xrightarrow{J_n^{op}} G^{op} \longrightarrow \text{Set}$$

- J_0^{op} is the graph
- J_1^{op} is the edges

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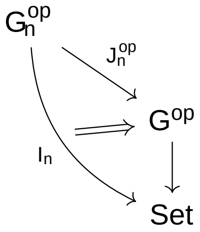

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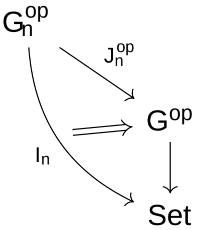

- 0-submorphisms are the subgraphs

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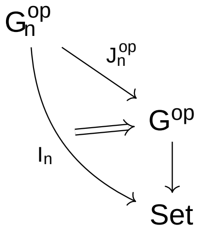

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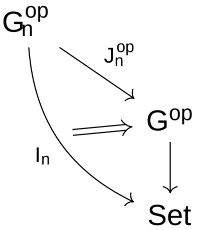

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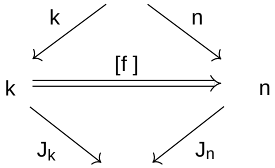

(for f : k ! n 2)

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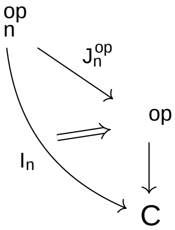

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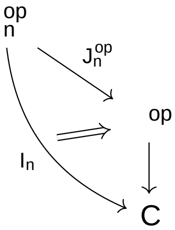

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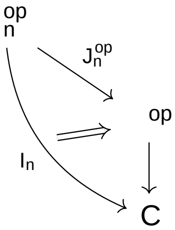

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$$\mathrm{dom}^0(S_1) \to \mathrm{sm}(\ )[0]$$

Formal Faces

for $f : k \to n \in$, we say S_k is an f-face of S_n if:

$$\begin{array}{ccc} n\,(S_n) & \xRightarrow{n\,(\;\,_n)} & n\,(J_n(\;)) \\ \Big\Downarrow i & & \Big\Downarrow\ (f\,) \\ k\,(S_k) & \xRightarrow{k\,(\;\,_k)} & k\,(J_k(\;)) \end{array}$$

Inuitively, this diagram is saying:
"Restrict the elements of S_n to the n-elements $^n \in S_n \cap \ [\ n]$ and check that $f\,(\ ^n) \in S_k$"

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$$
\begin{array}{ccc}
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\downarrow & & \downarrow k \\
(f\ )\ {}_1 n\ (S_n)=Q_k & \xhookrightarrow{U} & Q_k
\end{array}
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$$(\text{Where } Q_k = \ Cat(\ ; C)=k\ (J_k(\ ))$$

forgetting about the details: we get a subcategory

$$\hat{f}(S_n) \ ,! \quad sm(\ )[\ k]$$

This pullback allows us to prove basic properties of sm

sm( ) is a lax functor

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Proposition

sm( ) : ^{op} ! Cat_P is a Lax Functor

(Where Cat_P is the Kleisli Category)

sm is a functor

Moreover:

Theorem

sm : Fun(^{op}; C) ! Lax(^{op}; Cat_P) is a functor

That is, from any basic ontology ,
we get the "expansions", the submorphisms sm( )

Recap

- concepts are organized in a Basic Ontology ^{op} ! C

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$$O : \quad \to sm(T)$$

Recap

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Niavely an Ontological Expansion would be a natural

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Rigorous Ontological expansion: C = Set

However, : ^{op} ! Set

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However, : ^{op} ! Set, but sm(T) : ^{op} ! Cat$_P$

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Fr : Set ! Cat

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- tr$( ) = \quad$ f Fr $\quad$ g $: \quad ^{op} \; !$ Cat$_P$
- this gives us a (representable) functor

$$\text{tr} \; : \text{Fun}(\quad ^{op}; \text{Set}) \; ! \quad \text{Lax}(\quad ^{op}; \text{Cat}_P)$$

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In set, we have:

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Ontological Expansion in $C = \mathrm{Set}$

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- two basic ontologies $; T\ :\quad {}^{op}\ !\quad \mathrm{Set}$

Ontological Expansion

an <u>Ontological Expansion</u> is a natural transformation

$$O : \mathrm{tr}\,(\ )\quad !\quad \mathrm{sm}(T\,)$$

Current Work

What is $\mathrm{tr} : \mathrm{Fun}(\ ^{op}; C) \to \mathrm{Lax}(\ ^{op}; \mathrm{Cat}_P)$ for general C?

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Goal 3

Reconstruct data from it's elaborations

we're going to take cues from Grothendieck Topologies

Goal: Sheaf Theory of Ontologies

A grothendiek topology should be the rst example of an ontology

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- Category to organize concepts
- Coverings are elaborations
- Sheaf condition reconstructs data

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we want an O^0 O : tr ( ) ! sm(^{00})

Composition of expansions

Conjecture

sm: $(\quad{}^{op}{=}2\quad \mathrm{Cat})_{\mathrm{Lax}}\ !\ (\quad{}^{op}{=}2\quad \mathrm{Cat})_{\mathrm{Lax}}$

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Current Work

$$\mathrm{tr}\,(\ )\ \ !^{O}\ \mathrm{sm}(\ \ ^{0})\ !\ \ \mathrm{sm}(\mathrm{tr}\,(\ \ ^{0}))\ \ !^{\mathrm{sm}(O^{0})}\ \mathrm{sm}(\mathrm{sm}(\ \ ^{00}))\ !\ \ \mathrm{sm}(\ \ ^{00})$$

Current Work

$$\mathrm{tr}\,(\ )\ \xrightarrow{O}\ \mathrm{sm}(\ {}^{0})\ \to\ \mathrm{sm}(\mathrm{tr}\,(\ {}^{0}))\ \xrightarrow{\mathrm{sm}(O^{0})}\ \mathrm{sm}(\mathrm{sm}(\ {}^{00}))\ \to\ \mathrm{sm}(\ {}^{00})$$

this gives some conditions for tr

- we have a natural $l : \mathrm{tr}\,(\ {}^{0}) \to \mathrm{sm}(\mathrm{tr}\,(\ {}^{0}))$

Current Work

$$\mathrm{tr}(\;) \xrightarrow{O} \mathrm{sm}(\;^{0}) \to \mathrm{sm}(\mathrm{tr}(\;^{0})) \xrightarrow{\mathrm{sm}(O^{0})} \mathrm{sm}(\mathrm{sm}(\;^{00})) \to \mathrm{sm}(\;^{00})$$

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Current Work

$$tr(\ ) \xrightarrow{O} sm(\ ') \to sm(tr(\ ')) \xrightarrow{sm(O')} sm(sm(\ '')) \to sm(\ '')$$

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if this is the case, the ontological expansions are sm-algebras

$$[(\ ^{op}=2 \quad Cat)_{Lax}]_{sm}$$

Proposed De nition: Ontology

$$[(\quad ^{op} = 2 \quad \mathrm{Cat})_{\mathrm{Lax}}]_{\mathrm{sm}}$$

- Organize data: Objects are Basic Ontologies

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The condition that our ontological expansions compose gives us the following definiton:

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The condition that our ontological expansions compose gives us the following definiton:

Proposed Definition

A **Δ-Ontology** is a subcategory O , ! $[(\Delta^{op} {=}2 \quad \mathrm{Cat})_{Lax}]_{sm}$

References

intuition:
[OLOG] "OLOGS: A Categorical Framework for Knowledge Representation" David I. Spivak, Robert E. Kent

(higher) category theory:

[1] "Higher Categories, Higher Operads" Leinster
[2] "Higher Topos Theory" Lurie
[3] "Sheaves in Geometry and Logic" Maclane, Moerdijk
[4] "Categories for the working mathematician" Maclane
[5] "The Stack Of Microlocal Presheaves" Waschkies