Free Picard Categories

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  - $\pi_1(\mathscr{C}) = \mathscr{C}(I, I)$
  - $K : \pi_0(\mathscr{C}) \to \pi_1(\mathscr{C})$,
    $X \mapsto \beta_{X,X} \in \mathscr{C}(X \otimes X, X \otimes X) \cong \pi_1(\mathscr{C})$

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  - $(\beta : n + m \to m + n) = (-1_{n+m})^{nm}$

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The forgetful functor $U : \mathbf{Pic} \to \mathbf{Grpd}$ has a left adjoint given by

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Specifically: For $\mathscr{G} \in \mathbf{Grpd}$ and $\mathscr{A} \in \mathbf{Pic}$,

$$\mathbf{Pic}(\mathbb{Z}[\mathscr{G}], \mathscr{A}) \simeq \mathbf{Grpd}(\mathscr{G}, \mathscr{A})$$

as Picard categories, natural in $\mathscr{G}$ and $\mathscr{A}$.

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- Monoidal product: concatenation

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  - Braided hexagon
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$$\begin{array}{ccc} (n +_{\mathscr{Z}} n' +_{\mathscr{Z}} +n'')G & \xrightarrow{\delta} & (n +_{\mathscr{Z}} n')G + n''G \\ \delta \downarrow & & \downarrow \delta + \mathrm{Id} \\ nG + (n' +_{\mathscr{Z}} n'')G & \xrightarrow[\mathrm{Id}+\delta]{} & nG + n'G + n''G \end{array}$$

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- Note: $nG + (-n)G \cong (n - n)G = 0_{\mathbb{Z}} G \cong 0$

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  - $\overline{F}(-1_{1+n}.\mathrm{Id}_G) = K_{F(G)} + \sum_{|n|} \operatorname{sgn}(n) \mathrm{Id}_{F(G)}$

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  $\cong \sum_{|n|} \mathrm{sgn}(n) F(1.G) = \overline{{}_{\mathrm{u}}F}(n.G)$

Group rings and the free module

Group rings and the free module

Conjecture

For $\mathscr{G} \in \mathbf{Pic}$, $\mathcal{Z}[G]$ categorifies the group ring, in that

$$\begin{array}{ccc}
\mathbf{Pic} & \xrightarrow{\mathcal{Z}[_]} & \mathrm{CMon}(\mathbf{Pic}, *) \\
\pi_0 \downarrow \dashv \uparrow & & \pi_0 \downarrow \dashv \uparrow \\
\mathbf{Ab} & \xrightarrow[\mathbb{Z}[_]]{} & \mathrm{CMon}(\mathbf{Ab}, \otimes)
\end{array}$$

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and take cohomology of

$$\mathbb{Z}/2\text{-}\mathbf{Mod}\,(\mathcal{Z}[\mathbb{Z}/2], \mathcal{Z}) \to \mathbb{Z}/2\text{-}\mathbf{Mod}\,(\mathcal{Z}[\mathbb{Z}/2], \mathcal{Z}) \to \ldots$$

Thank you∼!

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